

ARO at ECMWF: current activities and future plans

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Acknowledge Katrin Lonitz



Outline

- **Current activities**

- We have not looked at/assimilated ARO yet in any detail
- Advised Pawel Hordyniec, Jennifer Haase, ... on how to adapt the existing two-dimensional observation operator in ROM SAF's ROPP software package (<https://www.romsaf.org/ropp/>)

- **Future plans**

- ECMWF is tasked with adding an ARO operator to ROPP-13 in 2025-26 as part of ROM SAF work

- **Aim to describe** how we currently forward model and assimilate *space based* GNSS-RO at ECMWF and outline how that approach can be adapted to ARO

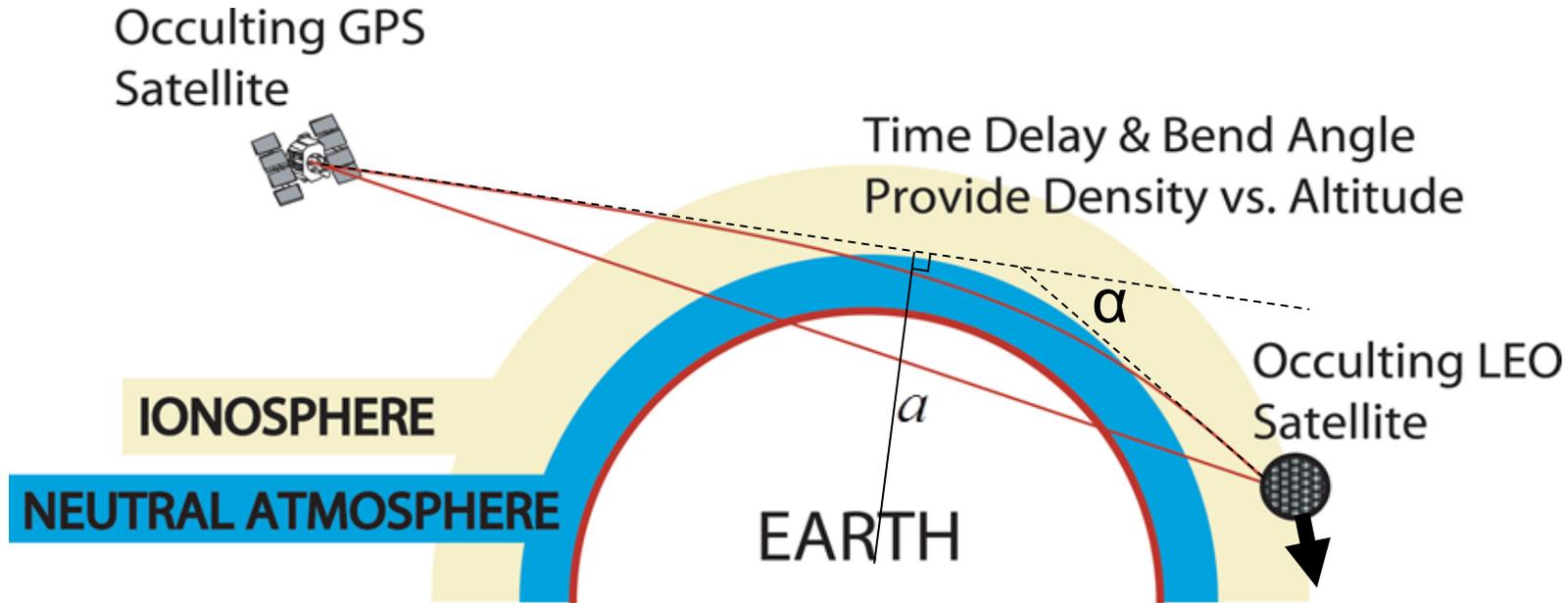
- Summary

To retrieve information from new observations in 4D-Var

- We need to provide
 - **The observation operator** (or forward model), $H(\mathbf{x})$, which maps the “state space” (**pressure, temperature, humidity, ... the variables we want to know**) to observation space
 - **An estimate of the observation error covariance matrix, \mathbf{R}** . This determines how closely we try to fit the observations – the “weight given to the observations” in the DA process.
 - **A quality control (QC) system**, to screen out gross measurement errors, that not consistent uncertainties values in \mathbf{R} . [NOT discussed here](#)

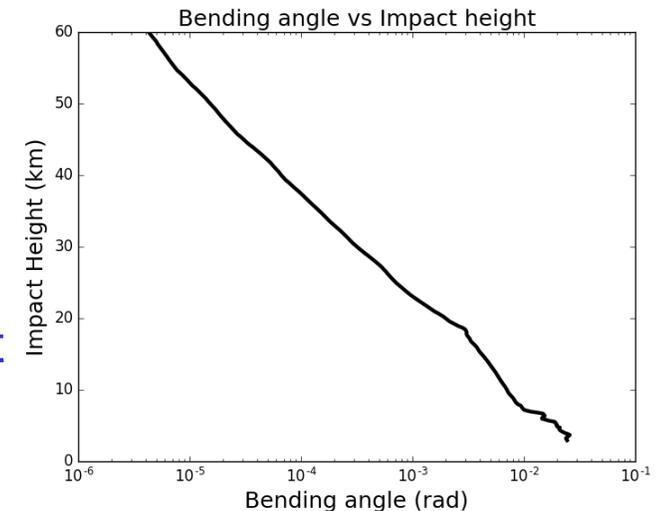
$$J(\mathbf{x}) = \frac{1}{2}(\mathbf{x} - \mathbf{x}_b)^T \mathbf{B}^{-1} (\mathbf{x} - \mathbf{x}_b) + \frac{1}{2}(\mathbf{y}_m - H(\mathbf{x}))^T \mathbf{R}^{-1} (\mathbf{y}_m - H(\mathbf{x}))$$

Space based GNSS-RO geometry



Setting occultation: as the LEO moves behind the earth we obtain a **profile** of bending angles, α , as a function of impact parameter, a .

The impact parameter is the distance of closest approach for the straight line path.



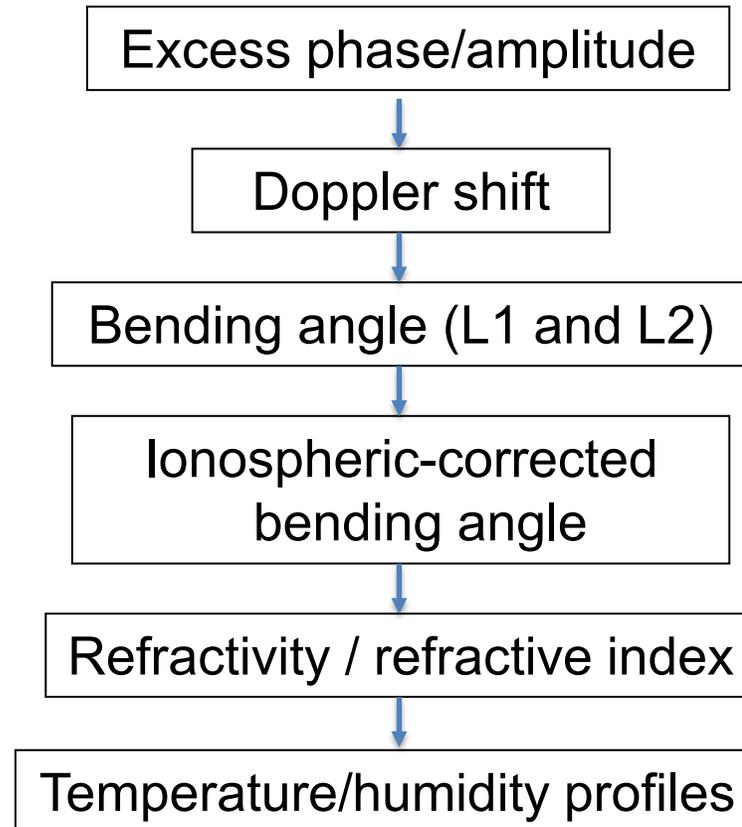
Why are GNSS-RO observations useful for NWP?

GNSS-RO complements the radiances!

Observations are useful if they provide **new** information.

- 1) GNSS-RO can be assimilated **without bias correction**. The observations are good for highlighting model errors/biases. Most satellite radiance observations require bias correction to the model. GNSS-RO measurements ***anchor the bias correction of radiance measurements***.
- 2) GNSS-RO (limb sounders in general) have **sharper weighting functions in the vertical** compared to radiances and therefore have good vertical resolution properties. The GNSS-RO measurements can “see” vertical structures that are in the **“null space”** of the satellite radiances.

Summary of GNSS-RO processing steps



Deriving the refractive index profiles

Assuming local **spherical symmetry**, we can use an **Abel transform** to retrieve a refractive index profile

Note the upper-limit of the integral!

$$n(x) = \exp\left(\frac{1}{\pi} \int_x^\infty \frac{\alpha(a)}{\sqrt{a^2 - x^2}} da\right)$$

The inverse Abel transform can be used to obtain the bending angle profile for a given refractive index profile (i.e. the inversion).

$$\alpha(a) = -2a \int_a^\infty \frac{d \ln n / dx}{\sqrt{x^2 - a^2}} dx$$

Corrected Bending angle as a function of impact parameter

Convenient variable ($x=nr$) (refractive index * radius)

Determining profiles of density (dry atmosphere only)

The refractive index (or refractivity) is related to the pressure, temperature and vapour pressure using two experimentally-determined constants (**from the 1950s and 1960s!**)

N= refractivity
n= refractive index
c1,c2 refractivity constants
P= pressure
T= Temperature
Pw= partial pressure of water vapour
ρ= density
R= gas constant

$$N = 10^6 (n - 1)$$
$$= \frac{c_1 P}{T} + \frac{c_2 P_w}{T^2}$$


If the water vapour is negligible, the 2nd term = 0, and the refractivity is proportional to the density

$$N \approx \frac{c_1 P}{T} = c_1 R \rho$$


So, although we don't know the values of P and T, we can use the ideal gas equation to retrieve a vertical profile of density!

“Dry temperature” retrieval

We need to estimate the temperature on a pressure level to integrate the hydrostatic equation

$$P(z) = P(z_u) - \frac{1}{c_1 R} \int_z^{z_u} N(z) g(z) dz$$

a priori

Then, the “**dry temperature**” can be calculated:

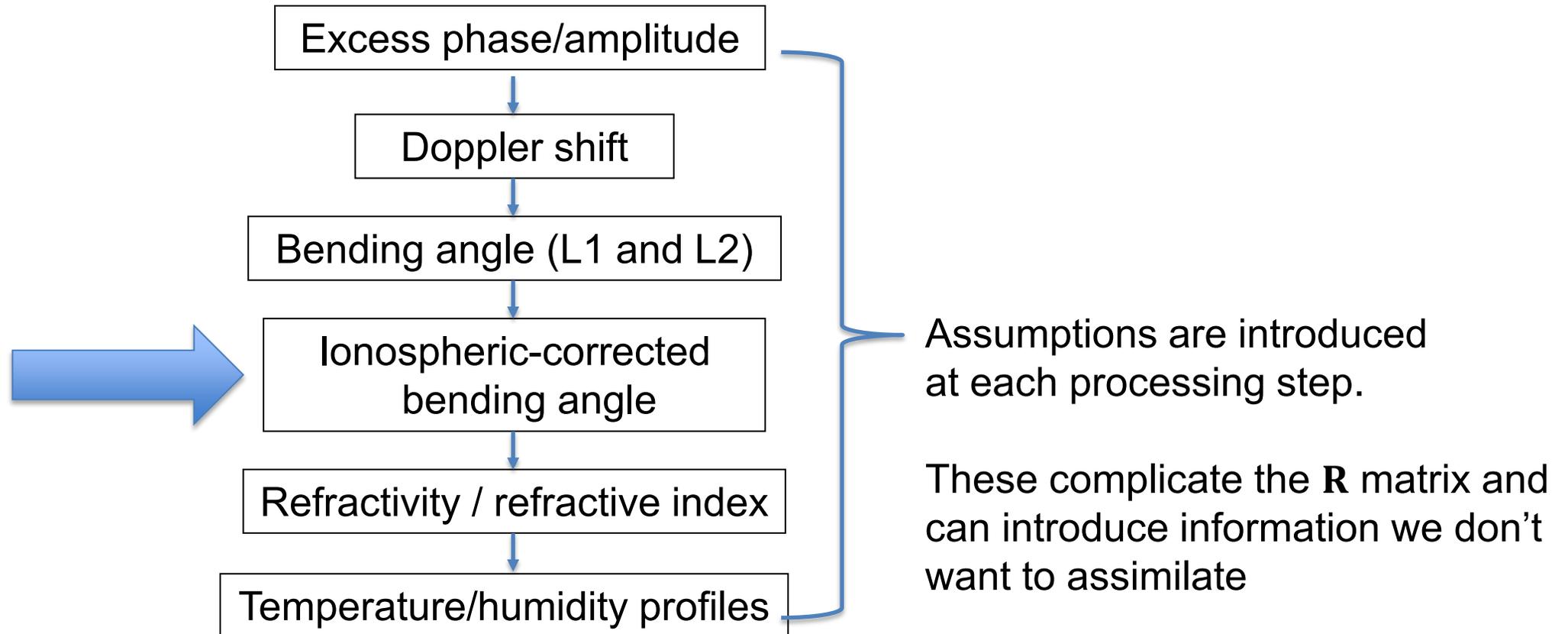
$$T(z) = c_1 \frac{P(z)}{N(z)}$$

Dry temperature is a horrible term that we use in RO. It’s a temperature retrieval that assumes humidity can be neglected

In the troposphere we need *a priori* information to partition the refractivity information into temperature and humidity

Summary of GNSS-RO processing steps

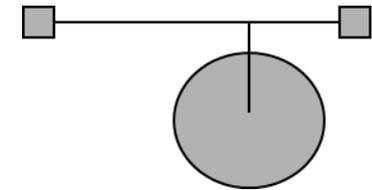
Which variable to assimilate?



GNSS-RO Assimilation with a 1D operator

- We assimilate bending angles with a 1D operator. **We ignore the real 2D nature of the measurement** and integrate

$$\alpha(a) = -2a \int_a^{\infty} \frac{d \ln n / dx}{\sqrt{x^2 - a^2}} dx$$

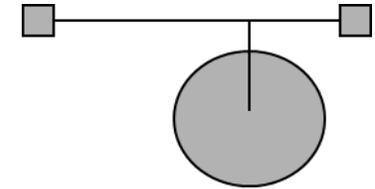


- The forward model is quite simple:
 - evaluate geopotential heights of model levels
 - convert geopotential height to geometric height and radius values
 - evaluate the refractivity, N , on model levels from P , T and Q using the refractivity coefficients. Compute $x = nr$.
 - Integrate, assuming refractivity varies exponentially between model levels.
- **We can introduce “tangent point drift”. Compute the integral at multiple locations as the tangent point moves in the horizontal**

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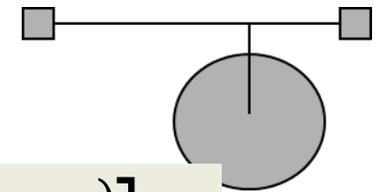
- The forward model is quite simple:
 - evaluate geopotential heights of model levels
 - convert geopotential height on model levels from P,T and to geometric height and radius values
 - evaluate the refractivity, N, Q using the refractivity coefficients. Compute $x = nr$.
 - Integrate, assuming refractivity varies exponentially between model level

We can introduce “tangent point drift”. Compute the integral at multiple locations as the tangent point moves in the horizontal

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$$\alpha(a) = -2a \int_a^{\infty} \frac{d \ln n / dx}{\sqrt{x^2 - a^2}} dx$$



$$\Delta\alpha = 10^{-6} \sqrt{2\pi a k_i} N_i \exp\{k_i(x_i - a)\} \left[\operatorname{erf} \left\{ \sqrt{k_i(x - a)} \right\} \right]_{x_i}^{x_{i+1}}$$

convert geopotential height on model levels from PT and to geometric height

Healy, S.B. and Thépaut, J.-.-N. (2006), Assimilation experiments with CHAMP GPS radio occultation measurements. Q.J.R. Meteorol. Soc., 132: 605-623. <https://doi.org/10.1256/qj.04.182>

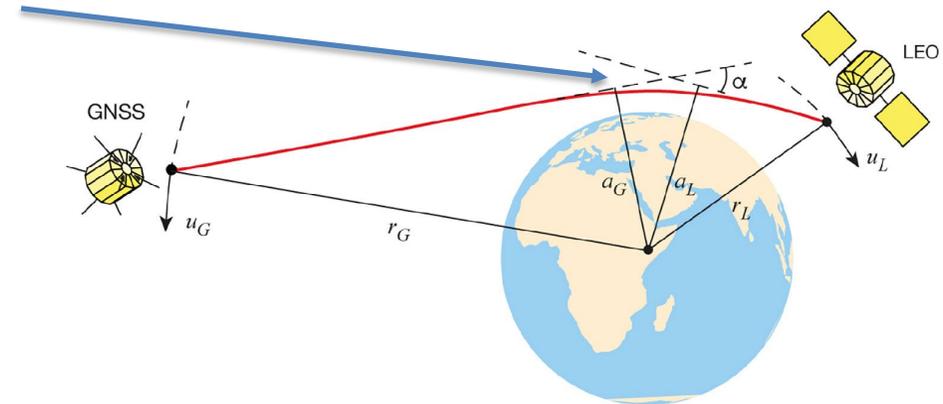
Burrows, C. P., et al : **Improving the bias characteristics of the ROPP refractivity and bending angle operators**, Atmos. Meas. Tech., 7, 3445-3458, doi:10.5194/amt-7-3445-2014, 2014.

Assimilating GNSS-RO with 2D operators

- We know that GNSS-RO is a limb measurement and the ray bending takes place over 100's of km in the horizontal
- Do we need to consider this when assimilating the data?
- Definition of the \mathbf{R} matrix used to assimilate the data
 - $\mathbf{R} = \mathbf{E} + \mathbf{F}$
 - \mathbf{E} is the instrument noise
 - \mathbf{F} is the forward model/representation error covariance matrix at accounts for simplifications in the in the forward model/scales that are represented in the model

The 2D operator should reduce the forward model errors

Tried by Zou *et al* in 1999



JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 104, NO. D18, PAGES 22,301–22,318, SEPTEMBER 27, 1999

A ray-tracing operator and its adjoint for the use of GPS/MET refraction angle measurements

X. Zou,¹ F. Vandenberghe,² B. Wang,^{1,3} M. E. Gorbunov,⁴ Y.-H. Kuo,² S. Sokolovskiy,⁴ J. C. Chang,⁵ J. G. Sela,⁶ and R. A. Anthes⁷

Move towards 2D GNSS-RO operators

The 2D operators take account of the real limb nature of the measurement, and this should reduce the forward model errors defined as

$$H(\mathbf{x}_t) - \mathbf{y}_t = \boldsymbol{\varepsilon}_f$$

Discrete representation of true state from model

Noise free observation

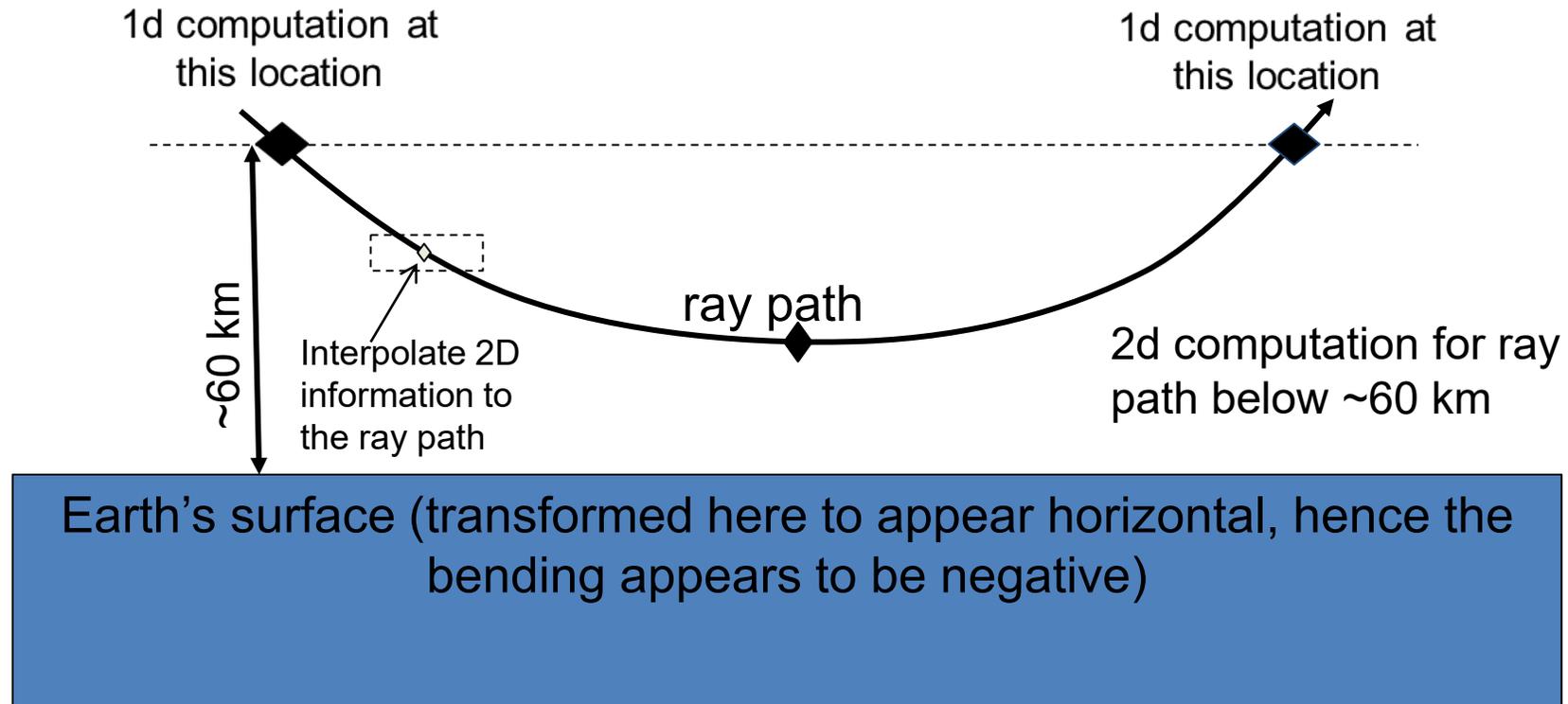
Forward model error

Reducing the forward model errors should improve our ability to retrieve information from the observation, but this must be balanced:

Extra Information versus **Additional Computing Costs**.

We are less likely to misinterpret information with a 2D operator

2D assimilation approach



Computational cost

Occultation plane described by 31 profiles with 40 km separation, spanning 1200 km

Assimilation with a 2D observation operator

Integrate these differential equations to determine the ray path in polar co-ordinates:

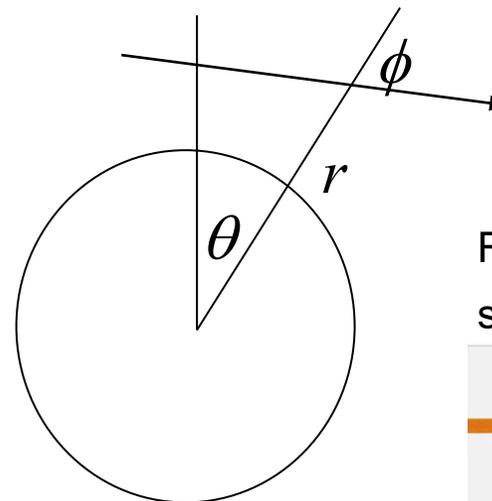
$$\frac{dr}{ds} = \cos \phi$$

$$\frac{d\theta}{ds} = \frac{\sin \phi}{r}$$

$$\frac{d\phi}{ds} \approx -\sin \phi \left[\frac{1}{r} + \left(\frac{\partial n}{\partial r} \right)_\theta \right]$$

They may look a bit daunting, but set $(\partial n / \partial r)_\theta = 0$ and they define a straight line!

$$\text{1D: } \alpha(a) = -2a \int_a^\infty \frac{d \ln n / dx}{\sqrt{x^2 - a^2}} dx$$

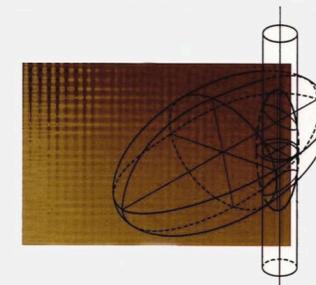


Rodgers, page 149

s = distance along ray path

Series on Atmospheric, Oceanic and Planetary Physics — Vol. 2

**INVERSE METHODS
FOR ATMOSPHERIC
SOUNDING**
Theory and Practice



Clive D. Rodgers

The R matrix

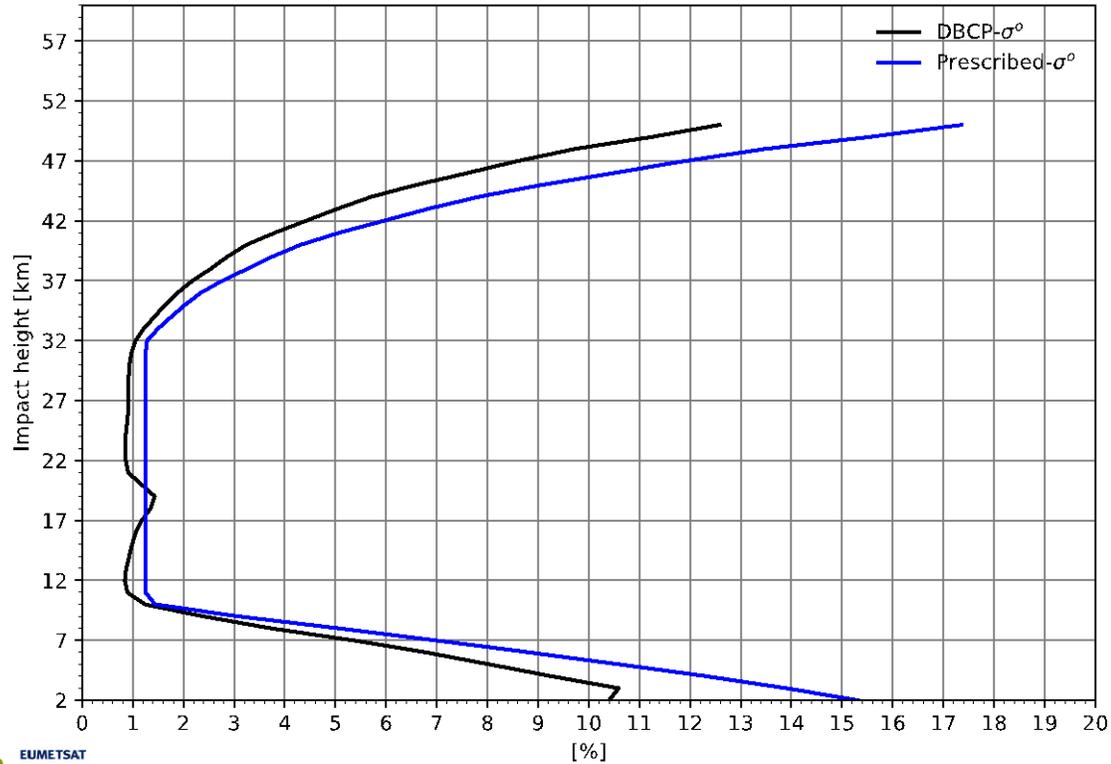
Assumed covariance matrix, \mathbf{R}

$$J(\mathbf{x}) = \frac{1}{2}(\mathbf{x} - \mathbf{x}_b)^T \mathbf{B}^{-1}(\mathbf{x} - \mathbf{x}_b) + \frac{1}{2}(\mathbf{y}_m - H(\mathbf{x}))^T \mathbf{R}^{-1}(\mathbf{y}_m - H(\mathbf{x}))$$

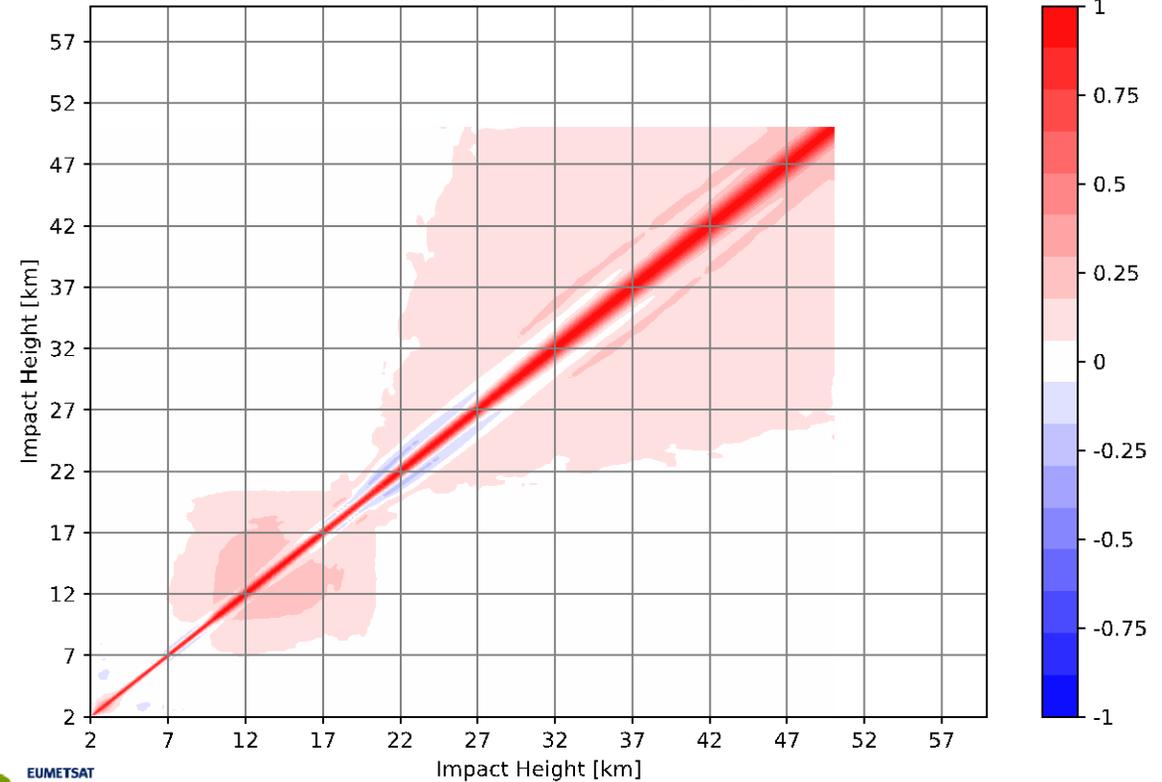
- The \mathbf{R} determines the weight we give to the GNSS-RO in the 4D-Var. We use a relatively crude global model at ECMWF. It is the same for all GNSS-RO instruments, and it ignores vertical error correlations
- We can compare the ECMWF uncertainty model with a “diagnosed” \mathbf{R} matrix – Desroziers, Three Cornered Hat. See, *for example*,
 - Todling, R, Semane, N, Anthes, R and S Healy (2022). The Relationship Between Two Methods for Estimating Uncertainties in Data Assimilation, QJRMS, available from: <https://doi.org/10.1002/qj.4343>

Comparing with diagnosed R matrix

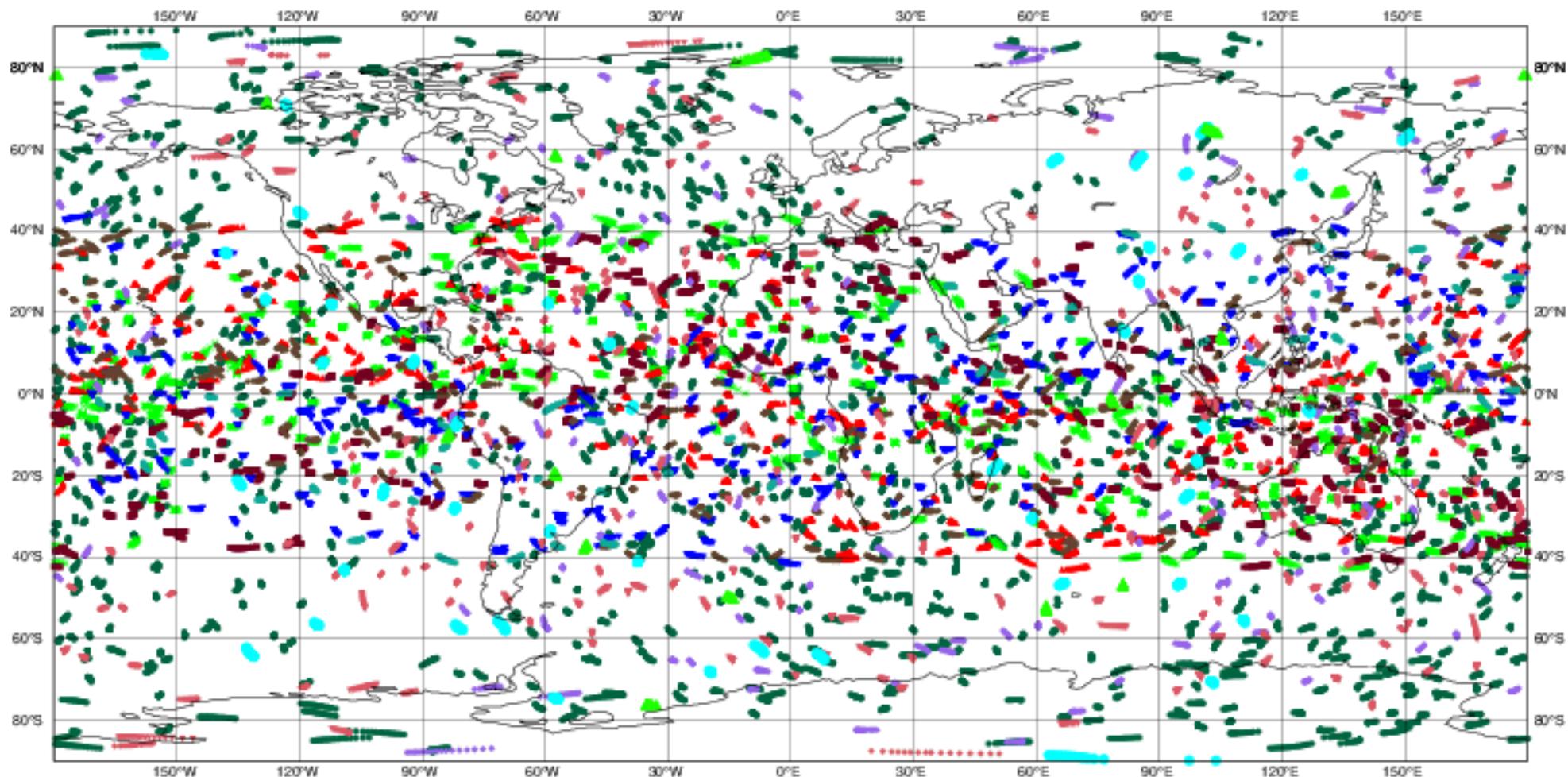
Uncertainty COSMIC-2_UCAR_May2023



Vertical error correlation COSMIC-2_UCAR_May2023



ECMWF data coverage (used observations) - GPSRO 2023062621 to 2023062703

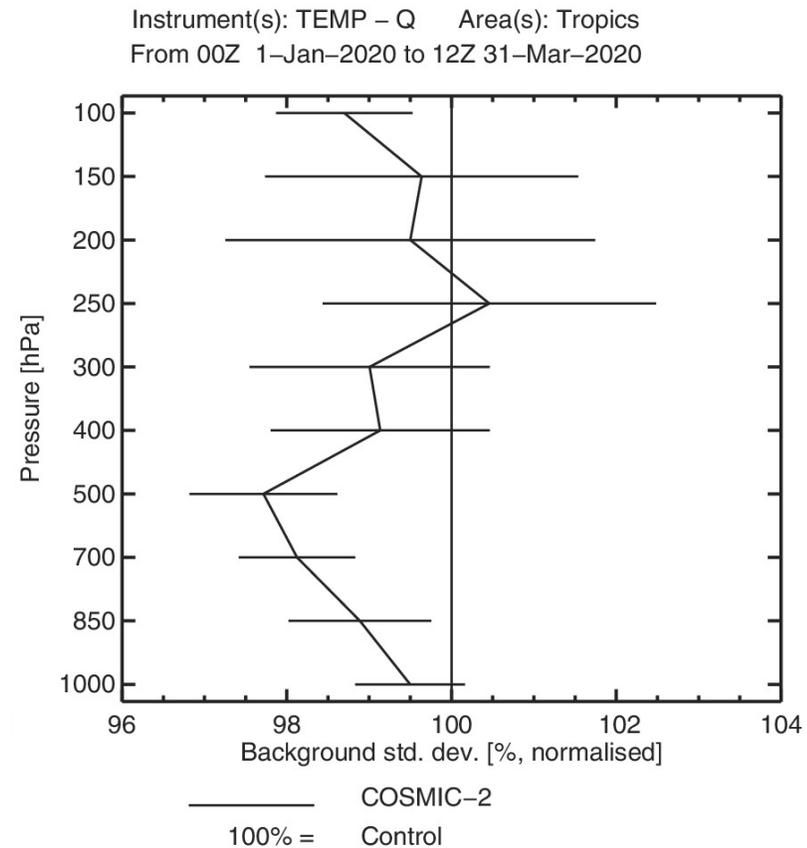


COSMIC-2 impact on tropospheric humidity

Ruston, B, Healy, S. Forecast Impact of FORMOSAT-7/COSMIC-2 GNSS Radio Occultation Measurements. *Atmos Sci*

Lett. 2021; 22:e1019. <https://doi.org/10.1002/asl.1019>

Improvement in
fit to sonde Q



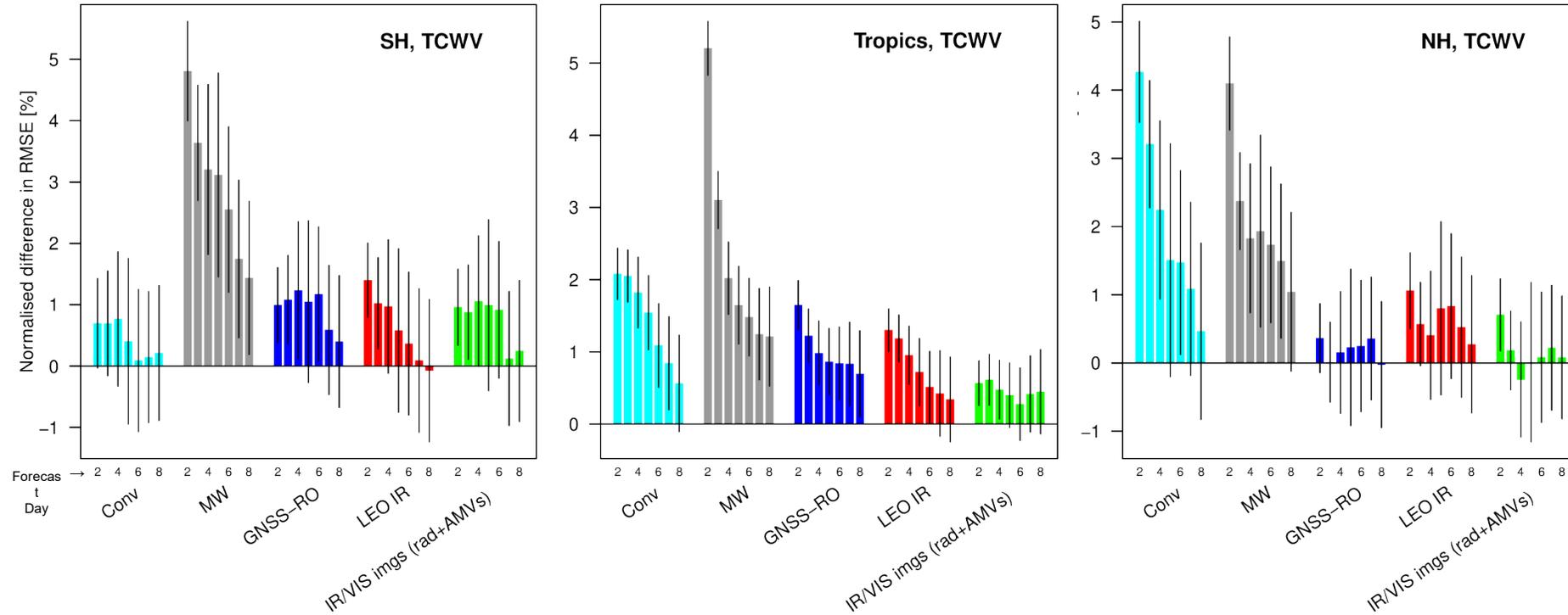
Impact of various observing systems at ECMWF

Provided by Niels Bormann – 2021 ECMWF annual seminar

https://events.ecmwf.int/event/217/contributions/2049/attachments/1397/2509/AS2021_Bormann.pdf

Bormann: Forecast impact, day 2-8: Total column water vapour

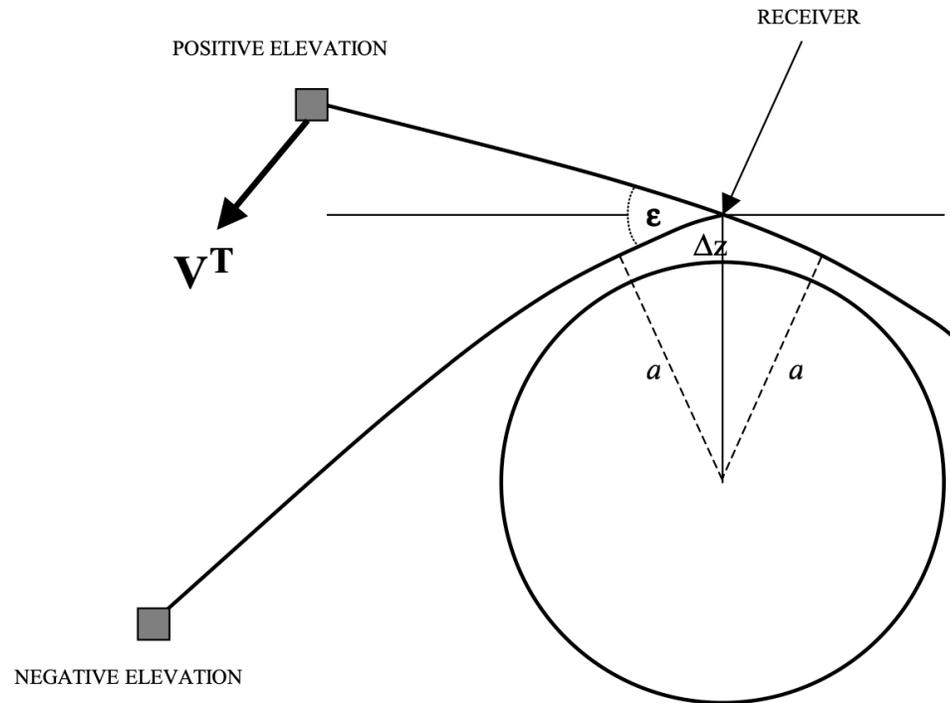
Verified against operational analyses, 3 periods combined



Letter to the Editor

Abel transform inversion of radio occultation measurements made with a receiver inside the Earth's atmosphere

S. B. Healy¹, J. Haase^{2,3}, and O. Lesne²



Define partial bending angle

$$\alpha'(a) = \alpha_N(a) - \alpha_P(a)$$

Subtracting the positive elevation b.a. from the negative elevation b.a. with the same impact parameter, a , gives the bending for path below the airborne receiver

Assuming spherical symmetry

Modified Abel transform pair (assuming spherical symmetry)

Partial bending angle
associated with path
below receiver

$$\longrightarrow \alpha'(a) = \alpha_N(a) - \alpha_P(a)$$

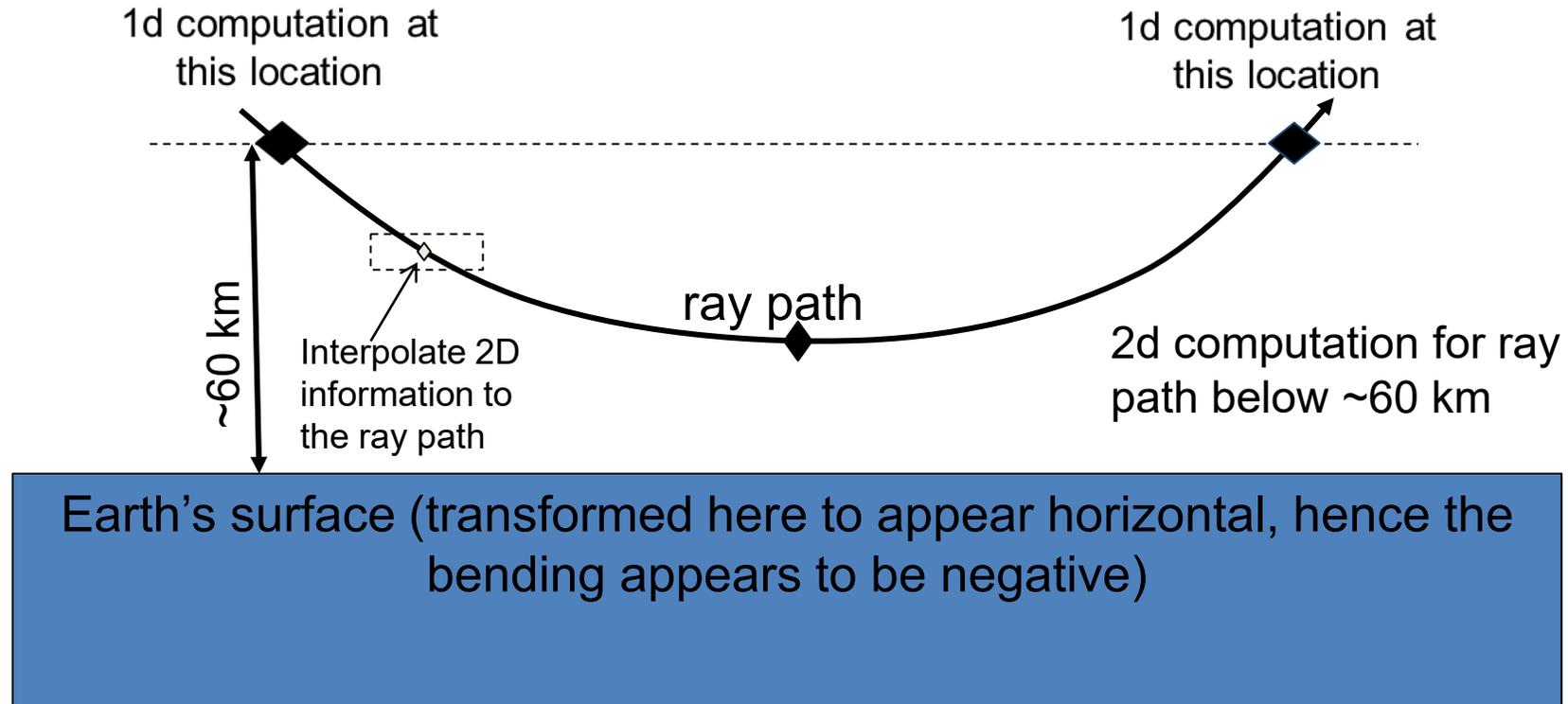
$$\alpha'(a) = -2a \int_a^{n^R r^R} \frac{\frac{d \ln n}{dx}}{(x^2 - a^2)^{1/2}} dx$$

$$n(x) = n^R \exp \left(\frac{1}{\pi} \int_x^{n^R r^R} \frac{\alpha'(a)}{(a^2 - x^2)^{1/2}} da \right)$$

Which variable to assimilate for ARO?

- Bending angle, $\alpha(a)$, is better than partial bending angle, $\alpha'(a)$
- The “partial bending” angle is only really useful to enable the Abel transform to refractivity
- In 4D-Var, we don’t need to “correct” an observation to conform to retrieval assumptions
- We should adapt the forward operator to conform to the measurement geometry

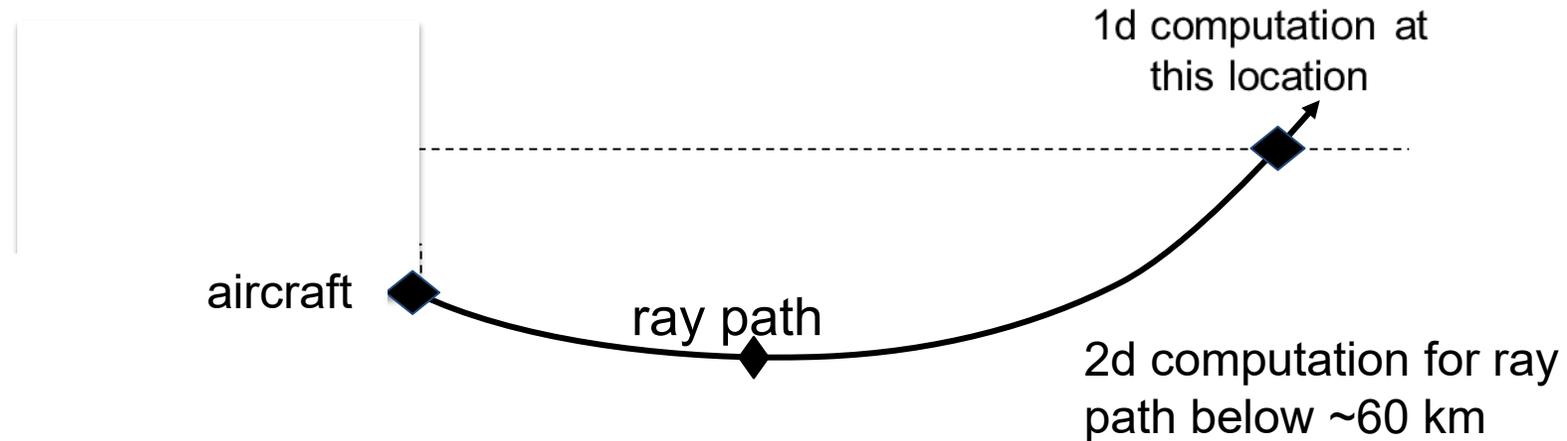
2D assimilation approach



Computational cost

Occultation plane described by 31 profiles with 40 km separation, spanning 1200 km

2D ARO assimilation approach



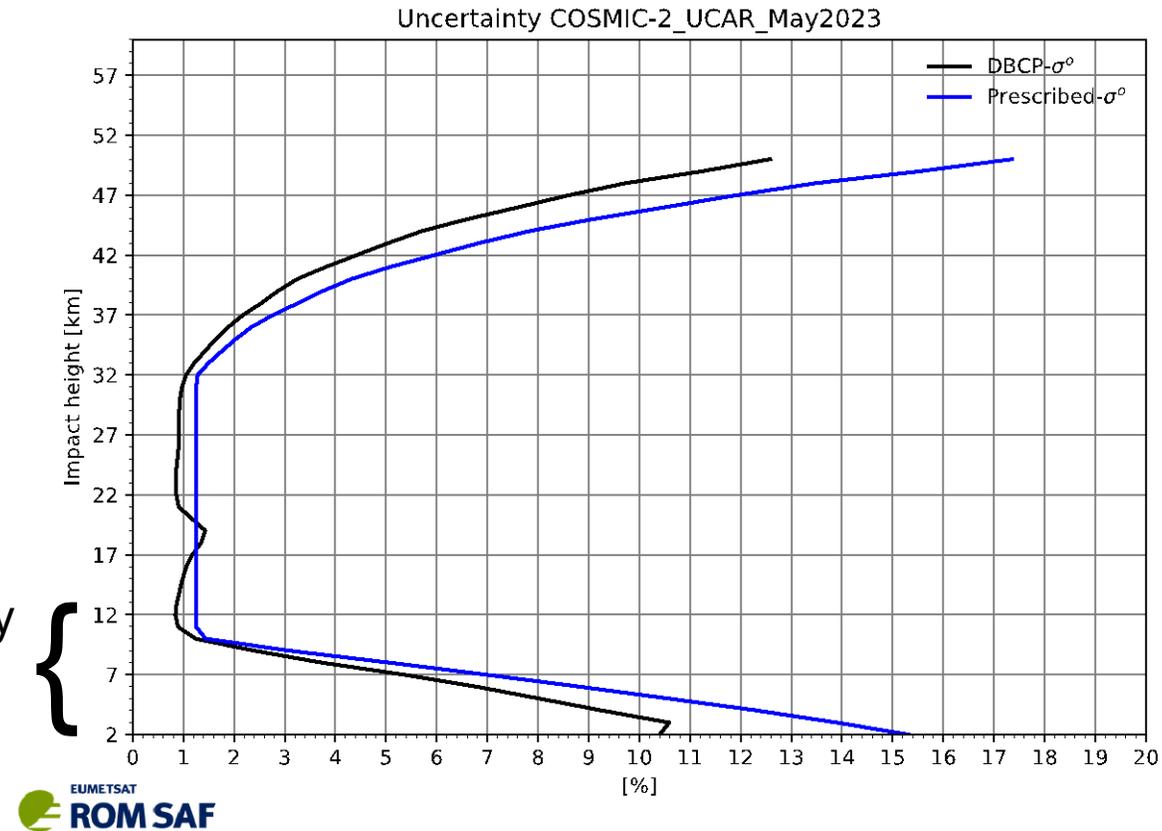
Earth's surface (transformed here to appear horizontal, hence the bending appears to be negative)

Computational cost

Occultation plane described by 31 profiles with 40 km separation, spanning 1200 km

ARO uncertainty

Is the ARO uncertainty similar to GNSS-RO here?



Summary

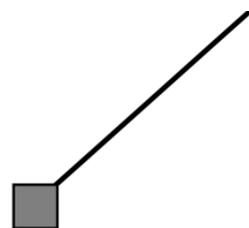
- ECMWF not really active in ARO at the moment, but has experience with GNSS-RO
- Plan to start working on ARO operator (TL,AD) for ROM SAF's ROPP-13 (~2024-5)
- Starting point is the similarity to assimilating space-based GNSS-RO
- Full bending angle better than partial bending angles for assimilation
- Essentially, adapt the current 2D operator to truncate the ray-path at the aircraft
 - Missing any subtleties here?
- Can I use the current GNSS-RO uncertainty model to assimilate ARO?

Letter to the Editor

Abel transform inversion of radio occultation measurements made with a receiver inside the Earth’s atmosphere

S. B. Healy¹, J. Haase^{2,3}, and O. Lesne²

$$\alpha'(a) = -2a \int_a^{n^R r^R} \frac{\frac{d \ln n}{dx}}{(x^2 - a^2)^{1/2}} dx$$



NEGATIVE ELEVATION

$$n(x) = n^R \exp \left(\frac{1}{\pi} \int_x^{n^R r^R} \frac{\alpha'(a)}{(a^2 - x^2)^{1/2}} da \right)$$

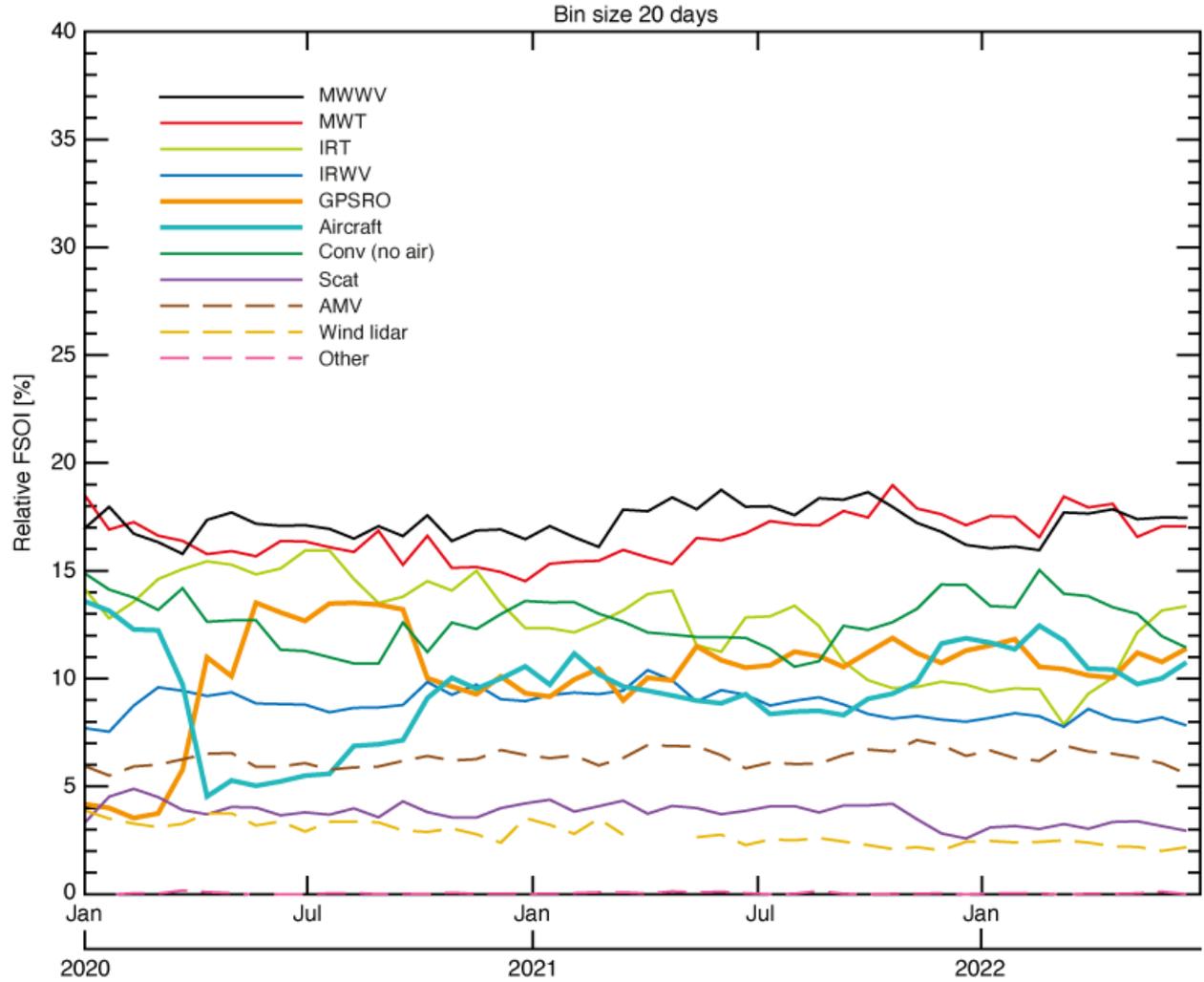
$$\alpha'(a) = \alpha_N(a) - \alpha_P(a)$$

Extra

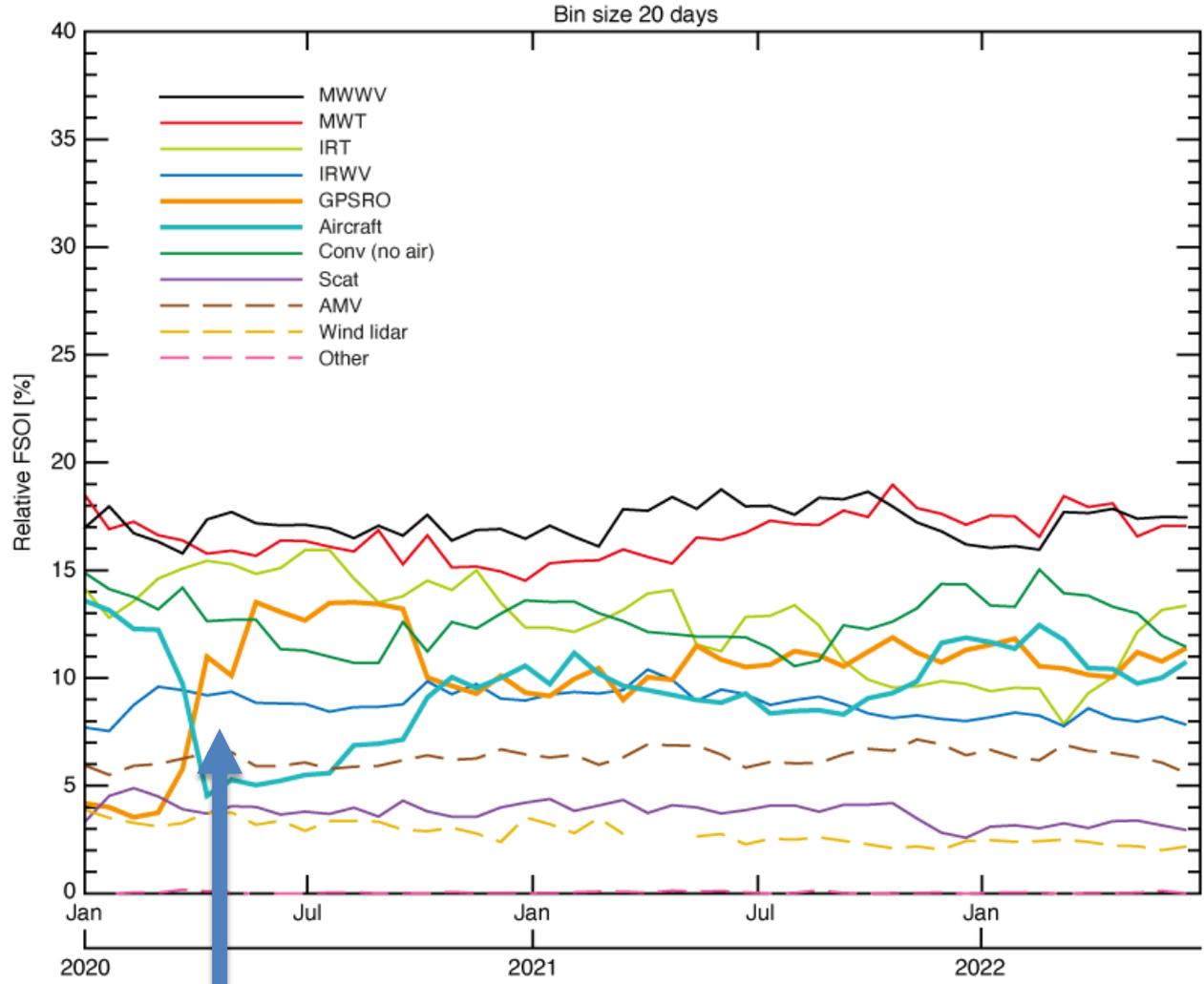
Forecast sensitivity to observation impact (FSOI)

- This is a **globally integrated scaler** quantity that summarises how observing systems contributed to the reduction of 24 hour forecast errors (surface pressure, wind, temperature). Theory outlined here
 - LANGLAND, R.H. and BAKER, N.L. (2004), Estimation of observation impact using the NRL atmospheric variational data assimilation adjoint system. *Tellus A*, 56: 189-201. <https://doi.org/10.1111/j.1600-0870.2004.00056.x>
 - Cardinali, C., 2009: Monitoring the observation impact on the short-range forecast. *Q. J. R. Meteorol. Soc.*, 135, 239-250, doi:10.1002/qj.366
 - Eyre, J.R., 2021: Observation impact metrics in NWP: A theoretical study. Part I: Optimal systems. *Q. J. R. Meteorol. Soc.*, 147, 3180-3200, doi:10.1002/qj.4123
- Usually provides information that is consistent with OSEs, **but can produce inconsistent results, where the FSOI of an observation type increases, but the quality of forecasts is degraded**
- Best to use FSOI alongside impact experiments

FSOI time series



FSOI time series



COSMIC-2

FSOI time series

