Coupled DA for interface observations

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Outline

- The coupled DA view of interface observations
- Using interface observations in the ECMWF variational DA system
- Applications
  - Skin temperature
  - Sea ice concentration
- Summary and Outlook

Interface observations in theoretical terms

$$y = \mathcal{H}(x_1, x_2) + \varepsilon$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Interface observations in ensemble methods

$$X_a = X_b + PH^T(HPH^T + R)^{-1}(y - HX_b)$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_a = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_b + PH^T(HPH^T + R)^{-1}(y - H\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_b)$$

Need to ensure that P appropriately represents cross covariances.

Need careful localization across components, as scales can change abruptly at an interface

Interface observations in variational DA with separate minimisations

$$J(\mathbf{x}) = \tfrac{1}{2}(\mathbf{x}-\mathbf{x}_b)^T B^{-1}(\mathbf{x}-\mathbf{x}_b) + \tfrac{1}{2}(y-\mathcal{H}(\mathcal{M}(\mathbf{x})))^T R^{-1}(y-\mathcal{H}(\mathcal{M}(\mathbf{x})))$$

$$J^b\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix}x_1-x_1^b\\x_2-x_2^b\end{bmatrix}^T B^{-1}\begin{bmatrix}x_1-x_1^b\\x_2-x_2^b\end{bmatrix}$$

$$J^b\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix}x_1-x_1^b\\x_2-x_2^b\end{bmatrix}^T \begin{bmatrix}B_{11} & B_{12}\\B_{21} & B_{22}\end{bmatrix}^{-1}\begin{bmatrix}x_1-x_1^b\\x_2-x_2^b\end{bmatrix}$$

$$J^b\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix}x_1-x_1^b\\x_2-x_2^b\end{bmatrix}^T \begin{bmatrix}B_{11} & 0\\0 & B_{22}\end{bmatrix}^{-1}\begin{bmatrix}x_1-x_1^b\\x_2-x_2^b\end{bmatrix}$$

$$J^b\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = \frac{1}{2}\left[x_1-x_1^b\right]^T\left[B_{11}\right]^{-1}\left[x_1-x_1^b\right] + \frac{1}{2}\left[x_2-x_2^b\right]^T\left[B_{22}\right]^{-1}\left[x_2-x_2^b\right]$$

$$J^b\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = J_1^b(x_1) + J_2^b(x_2)$$

Interface observations in variational DA with separate minimisations

$$J(\mathbf{x}) = \frac{1}{2}(\mathbf{x}-\mathbf{x}_b)^T B^{-1}(\mathbf{x}-\mathbf{x}_b) + \frac{1}{2}(y-\mathcal{H}(\mathbf{x}))^T R^{-1}(y-\mathcal{H}(\mathbf{x}))$$

$$J_1^o(x_1) = (y_1 - \mathcal{H}_1(x_1))^T R_1^{-1}(y_1 - \mathcal{H}_1(x_1))$$

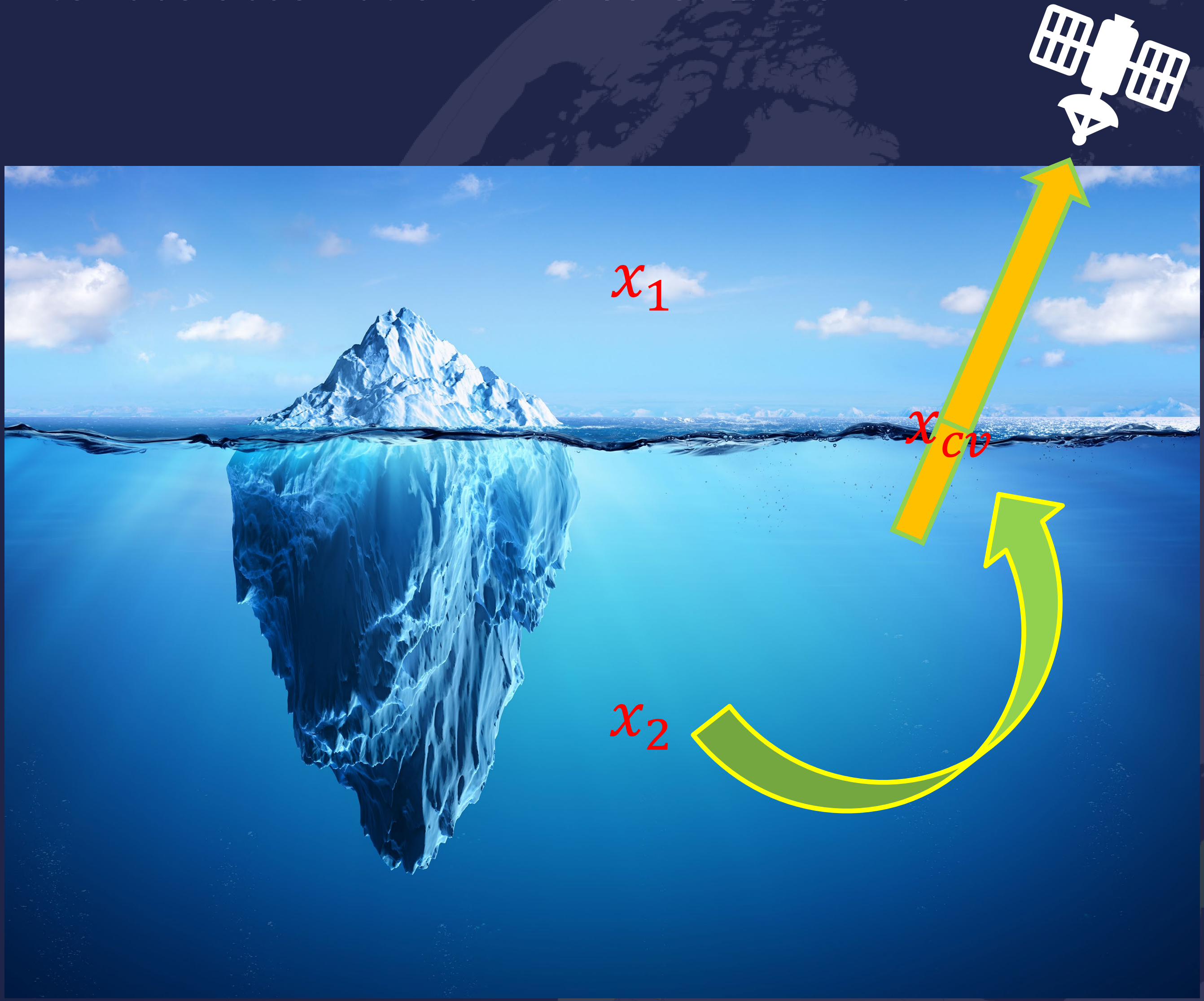
$$J_2^o(x_2) = (y_2 - \mathcal{H}_2(x_2))^T R_2^{-1}(y_2 - \mathcal{H}_2(x_2))$$

Minimising $J_1(x_1)$ and $J_2(x_2)$ then minimises $J(\mathbf{x})$

What about these pesky interface observations??

$$J_c^o(x_1, x_2) = (y_c - \mathcal{H}_{12}(x_1, x_2))^T R_c^{-1}(y_c - \mathcal{H}_{12}(x_1, x_2))$$

Interface observations in theoretical terms

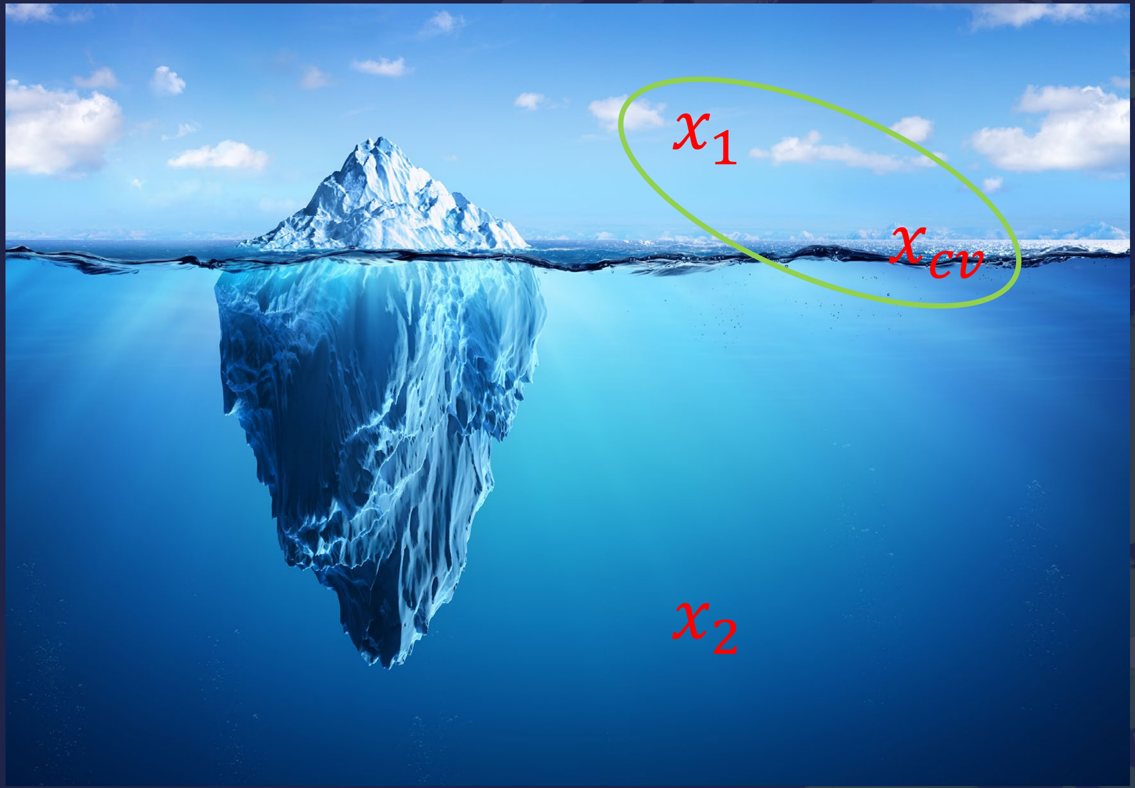


$$y_c = \mathcal{H}_{12}(x_1, x_2) + \varepsilon$$

$$\mathcal{H}_{12}(x_1, x_2) = \mathcal{H}_c(x_1, x_{\mathrm{cv}}) = \mathcal{H}_c(x_1, f(x_2))$$

$$x_{cv} := f(x_2)$$

Interface observations in theoretical terms



x_{cv} is an eXtended control variable

$$J_1(x_1, x_{cv}) = J_1^b + J_1^o + J_c^o$$

$$J_2(x_2) = J_2^b + J_2^o + J_{\mathrm{cv}}$$

$$J_{\mathrm{cv}} := (x_{cv} - f(x_2))^T \Xi^{-1} (x_{cv} - f(x_2))$$

Implementation of the J_{cv} term

How the J_{cv} term is implemented depends on how the x_{cv} term is implemented

$$J_{\mathrm{cv}} := (x_{cv} - f(x_2))^T \Xi^{-1} (x_{cv} - f(x_2))$$

If the x_{cv} is a 2-D field (i.e. in model space), f is a function of a projection of x_2, and so

$$J_{\mathrm{cv}} = (x_{cv} - \mathcal{P}(x_2))^T B_{\mathrm{cv}}^{-1} (x_{cv} - \mathcal{P}(x_2))$$

To implement this you have to consider the preconditioning of the variational solver.

However if the x_{cv} is a collection of 0-D points (i.e. in observation space):

$$J_{\mathrm{cv}} = (x_{cv} - \mathcal{H}_c(x_2))^T R_{\mathrm{cv}}^{-1} (x_{cv} - \mathcal{H}_c(x_2))$$

where **f** is now a (new) observation operator.

These can be easily added as observations, weighted by an observation error covariance

Sebastien will describe the xcv in more detail shortly

Skin temperature

$$x_{cv} = T_{\text{skin}} = T_{0.5m} + \delta_{cs}$$

$$\frac{\partial x_{cv}}{\partial x_2} = \frac{\partial T_{0.5m}}{\partial x_2} + \frac{\partial \delta_{cs}}{\partial x_2}$$

$$\frac{\partial x_{cv}}{\partial x_2} \approx H_{sst} + 0$$

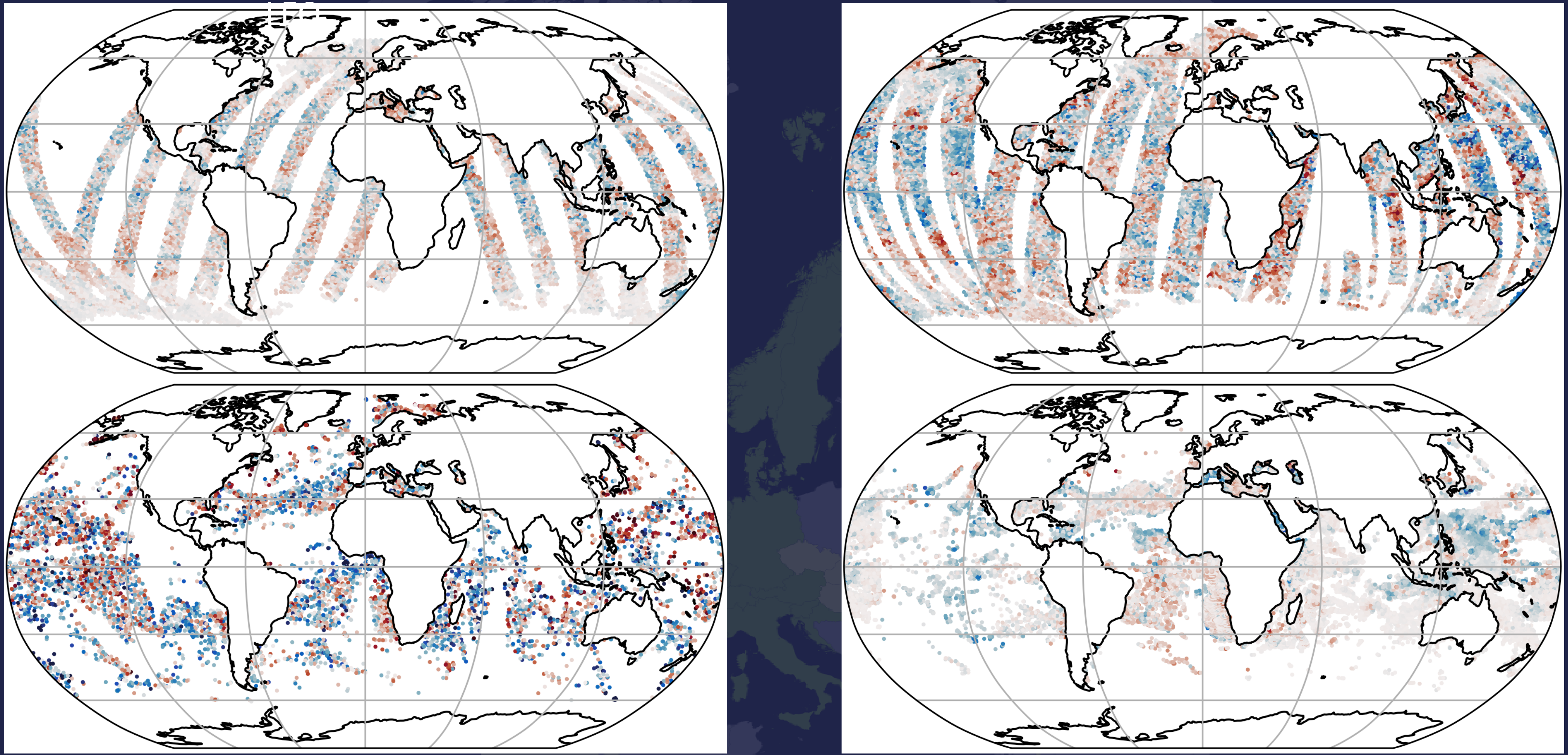
Skin temperature

$$x_{cv} - \mathcal{H}_c(x_2)$$

See Tracy Scanlon’s talk on Thursday

GMI: MW

AMSR2: MW LEO

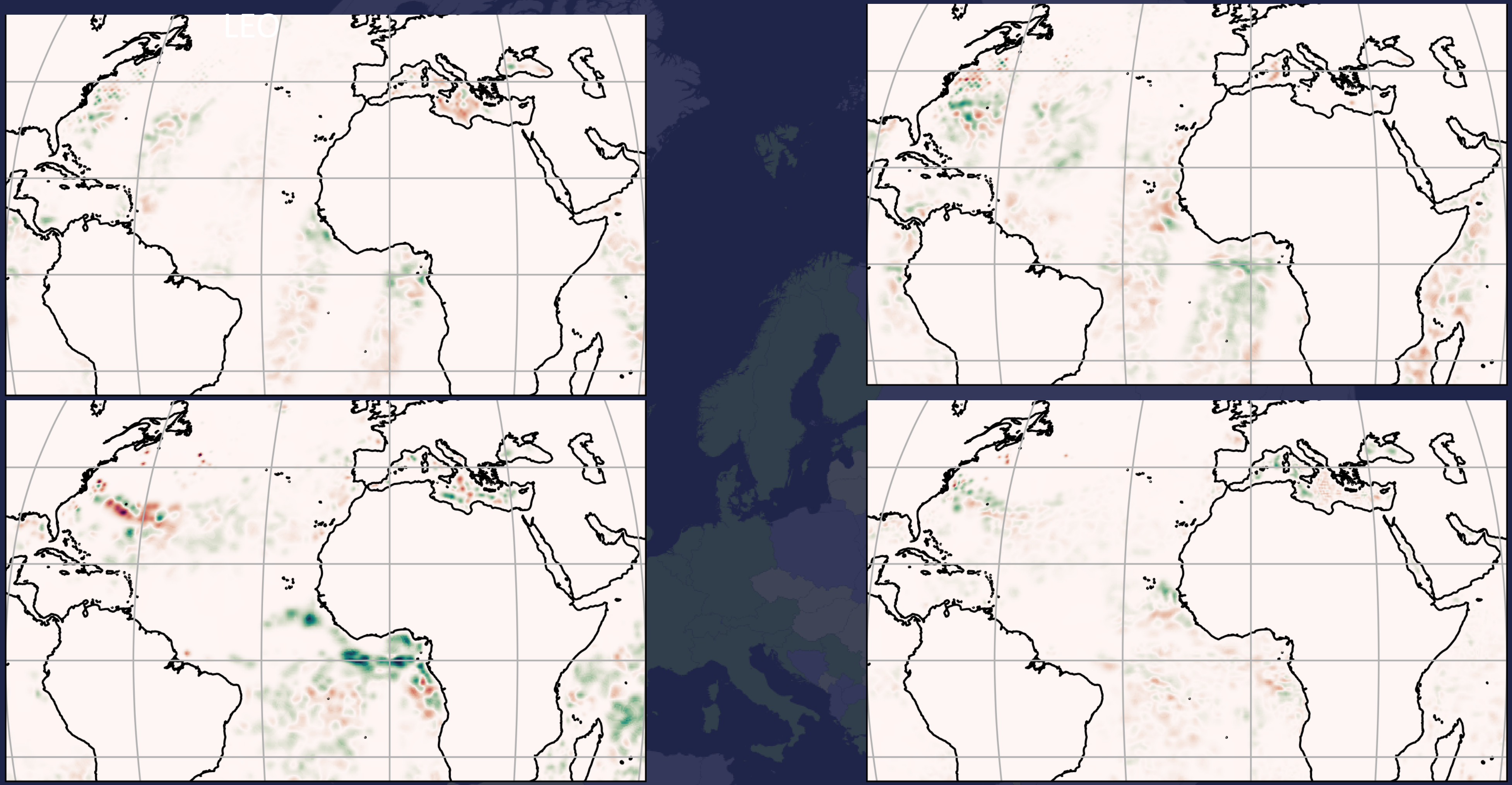


CrIS: Hyperspectral IR LEO

IR: GEOS Meteosat/GOES/Himawari

Skin temperature

$\delta x|_{SSH}$



Sea ice concentration

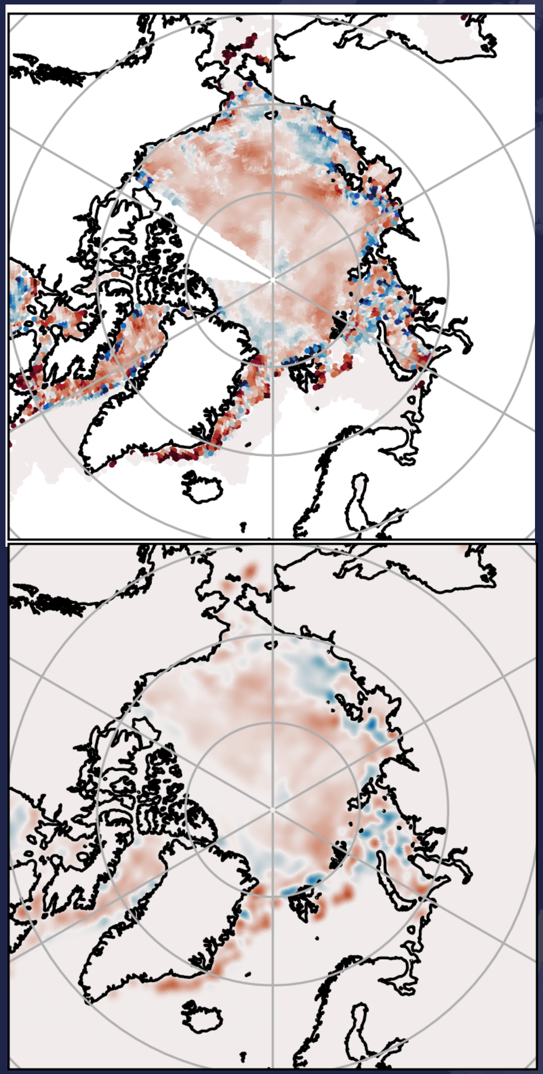
$$x_{cv} = SI_{eff} = SI_{conc} - MP_{conc}$$

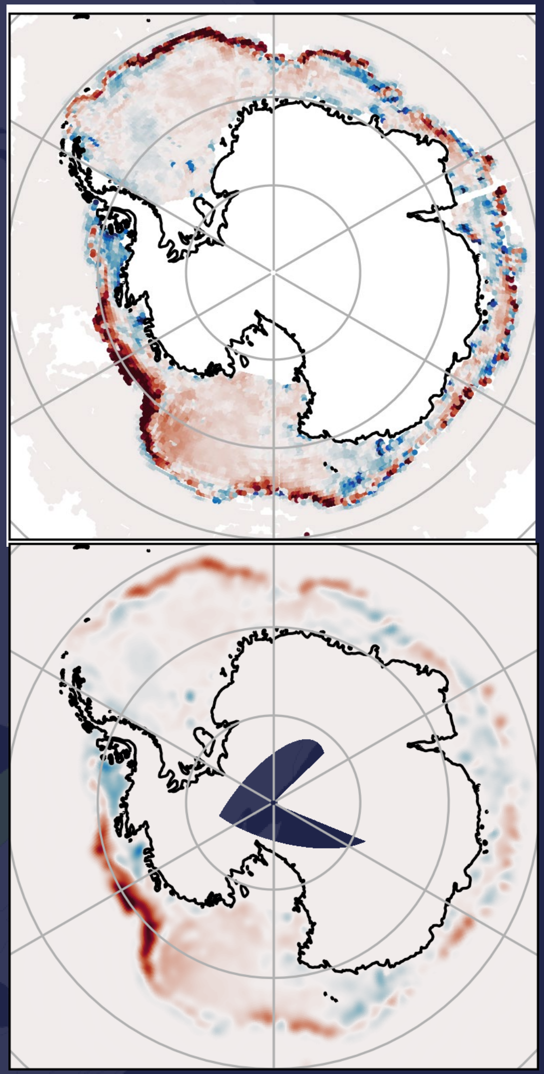
$$\frac{\partial x_{cv}}{\partial x_2} = \frac{\partial SI_{eff}}{\partial x_2} = H_{\mathrm{sic}}$$

See Alan Geer's talk on Wednesday

Sea ice concentration

$$x_{cv} = SI_{eff} = SI_{conc} - MP_{conc}$$





Summary and Outlook

- Interface observations have to be treated with care in the coupled DA context
- Can fit interface observations into coupled variational DA with an
  - Extended control vector, and
  - A penalty term to constrain the ocean/sea ice to the extended control vector

- SKT and SIC is working well
  - Instrument specific
  - Can continue to be refined and optimized

- Promising results on using altimeter range observations in the same framework
  - Flexible system which can be implemented easily for new observation types
- Need to introduce further in situ observations for independent verification and future anchoring assimilation